Phi_TS function

The Phi_TS function is useful for calculating thermal stress or deflection in problems in which a rod experiences a temperature gradient along its axis, as discussed in *Cryogenic Heat Transfer* by Barron and Nellis (2016). The function is defined as:

$$\Phi(T) = \int_{T_0}^{T} \left[\frac{\Delta L}{L}(T) - \frac{\Delta L}{L}(T_0) \right] k_t(T)\, dT$$

where T_0 is the initial temperature of the material. The end deflection of the rod (δ) may be written as follows:

$$\delta = \left(\frac{\sigma}{E}\right) L + \frac{(\Phi_H - \Phi_C)}{(K_H - K_C)} L$$

where σ is the applied stress, E is Young's modulus, L is the length of the rod, Φ_H and Φ_C are the values of Φ evaluated at the rod end temperatures (T_H and T_C), K_H and K_C are the integrated conductivity functions evaluated at T_H and T_C.

If both ends of the rod are rigidly fixed such that the deflection is zero, (i.e., the rod is externally constrained), then the mechanical deflection is zero and the resulting stress is obtained from.

$$\frac{\sigma_{th}}{E} = -\frac{(\Phi_H - \Phi_C)}{(K_H - K_C)}$$

If both ends of the rod are free then the applied mechanical stress is zero and the deflection can be obtained from:

$$\delta = \frac{(\Phi_H - \Phi_C)}{(K_H - K_C)} L$$

The calling protocol for the function is:

Phi_TS(T, T_0, S$)

where:
T is the temperature to use as the upper limit of the integral (K, R, F, or C)
T_0 is the temperature to use as the lower limit of the integral, the initial temperature (K, R, F, or C)
S$ is a string containing the substance of interest (which must be an incompressible substance in the EES database)

The function returns the integral defined above (W/m or Btu/hr-ft).

The example below computes the thermally induced stress in a bar of 304 Stainless Steel that is initially at 300 K when its ends are constrained and one end is held at 300 K while the other is held at 150 K. The result is 2.016×10^8 Pa (201.6 MPa).

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$UnitSystem SI Mass J Pa K

T_0=300 [K]                                   "initial temperature"
T_H=300 [K]                                   "hot end temperature"
T_C=150 [K]                                   "cold end temperature"
S$='Stainless_AISI304'                        "material"

K_H=IntK(S$,T=T_H)                            "Integrated conductivity at T_H"
K_C=IntK(S$,T=T_C)                            "Integrated conductivity at T_C"
Phi_H=Phi_TS(T_H,T_0,S$)                      "Phi function at T_H"
Phi_C=Phi_TS(T_C,T_0,S$)                      "Phi function at T_C"
E=YoungsModulus(S$, T=T_0)*Convert(GPa,Pa)    "Young's modulus"
sigma_th=-E*(Phi_H-Phi_C)/(K_H-K_C)           "thermal stress"
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